CORRESPONDENCE

A SIMPLIFIED EQUATION FOR MINIMUM TEMPERATURE PREDICTION

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An invaluable aid in the quantitative prediction of radiation frost is an equation which expresses the minimum temperature as a function of the evening surface hygrometric data. Many such equations have been developed empirically for application to the source region of the original data (c. f. [1, 2]). Because the net loss of heat from the ground at night depends on complex radiation and heat conduction processes (c. f. [3, 4]), the weakness of any empirical equation based on surface data is the assumption of average values of several physical parameters which are lumped together as coefficients in the empirical equation. In certain climatic zones, however, particularly in the low-level valleys of California, this has proved to be a valid assumption. The purpose of this note is to add a very simple formula to the large family of empirical formulas that have been published previously in the *Review* (e. g., [1, 2, 5, 6]).

The formula devised by Young [7] has been adapted for use in the orange belt of the southern San Joaquin Valley of California [6]; in the majority of cases the indicated minimum is in error by less than $\pm 3°$ F. The formula as currently used is as follows:

$$T_m = D - \left(\frac{H-30}{4}\right) + v + v' \tag{1}$$

where T_m is the indicated minimum temperature (° F.)
D is dewpoint (° F.) at 4:45 p. m. PST
H is relative humidity (percent) at 4:45 p. m. PST
v and v' are variable corrections that are functions of D and H, respectively. These functions, to an approximation that is within the error mentioned, are linear equations:

$$v = -\left(\frac{D-28}{3}\right) \text{ and } v' = \frac{H-52}{6} \tag{2}$$

Substitution of (2) into (1) simplifies the prediction formula to:

$$T_m = \frac{2D}{3} + \frac{98-H}{12} \tag{3}$$

Although this formula is a useful simplification of equation (1), its application with the directly observed variables, dry-bulb temperature T_d and wet-bulb temperature T_w, requires the use of psychrometric tables to obtain D and H. Thus, further simplification would result if the prediction formula was expressed directly as a function of T_d and T_w. Such a prediction formula of the following form was indeed proposed long ago by Ångström [8]:

$$T_m = a(T_w - k) - bT_d \tag{4}$$

where a, k, and b are constants to be determined empirically. Ångström found $a \approx 1$ and b to be so small that the last term could be dropped; thus his prediction formula reduced to

$$T_m = T_w - k \tag{5}$$

As prediction formulas of the form of (5) generally have not been found to be as successful as Young's formula (c. f. [2]), it seemed desirable to attempt to derive from his formula a new formula with both T_w and T_d as independent variables. To develop such a relation, hypothetical minima T_m were calculated from Young's formula for such ranges of temperature and dewpoint as usually precede frosts in the San Joaquin Valley. Examination of the results showed that for a given value T_d, the range of $T_w - T_m$ did not exceed 3.5° F.; thus, with small error, this difference could be taken as constant. Figure 1 shows the values of $T_w - T_m$ from the original work sheet plotted against T_d. The straight line, which closely fits the plotted points, gives the following simplified prediction formula:

$$T_m = T_w - \frac{1}{4}(T_d + 16) \tag{6}$$

This simple hygrometric equation, which was devised in 1942 and has been tested since that time in the Lindsay, Calif., and the Modesto, Calif., fruit-frost districts, gives accurate results in those areas on clear, quiet nights of winter and spring. Actual comparison with the original Young formula over the entire range of temperature and dewpoint (nights with frost only) shows an average difference in the indicated minimum of less than $\pm 0.5°$ F.

The advantages of the simplified equation (6) are twofold: it is not necessary to refer to tables for dewpoint and relative humidity values, or to tables of constants—the indicated minimum can be calculated mentally and quickly when the dry and wet bulb temperatures are known;

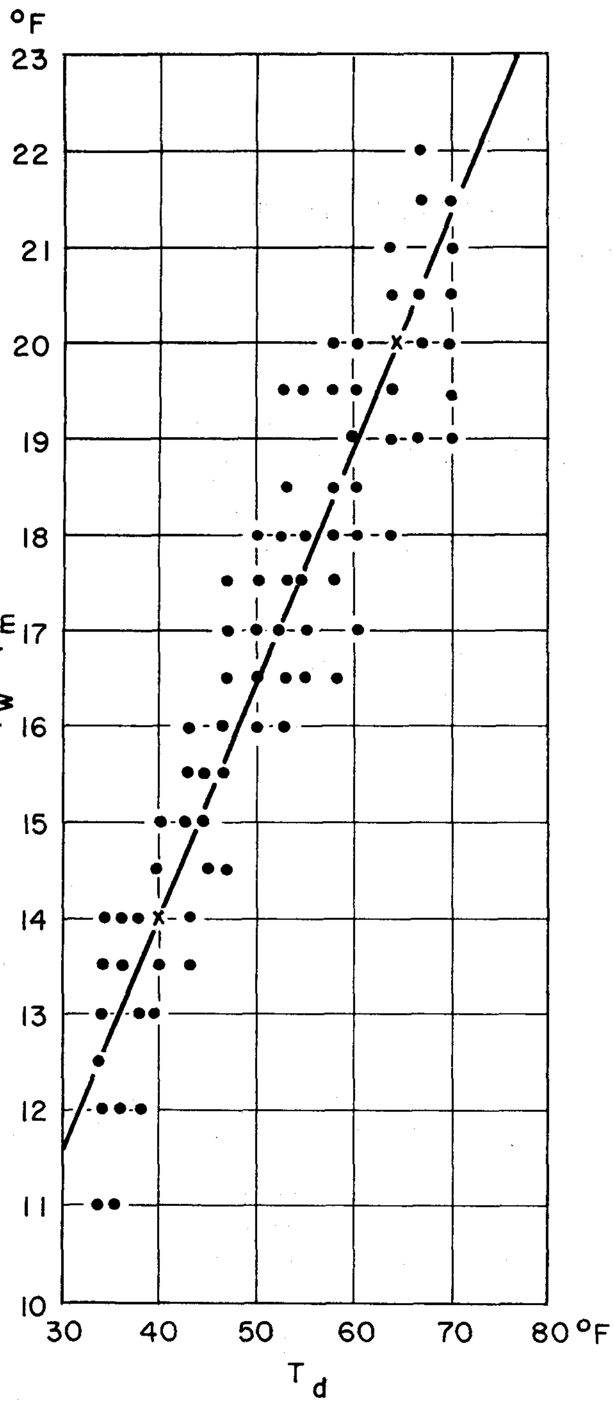

FIGURE 1.—Relation between dewpoint temperature at 4:45 p. m. PST and the difference $(T_w - T_m)$ for the San Joaquin Valley.

further, the tests in the Lindsay and Modesto districts suggest that this form of equation can easily be fitted to data for climatically similar localities by changing the slope and intercept constants. It would be of interest to plot the relation of $(T_w - T_m)$ vs. T_d in climatically different localities to test whether the linearity is of a local or universal nature.

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